Understanding the Signal to Noise Ratio in Design of Experiments

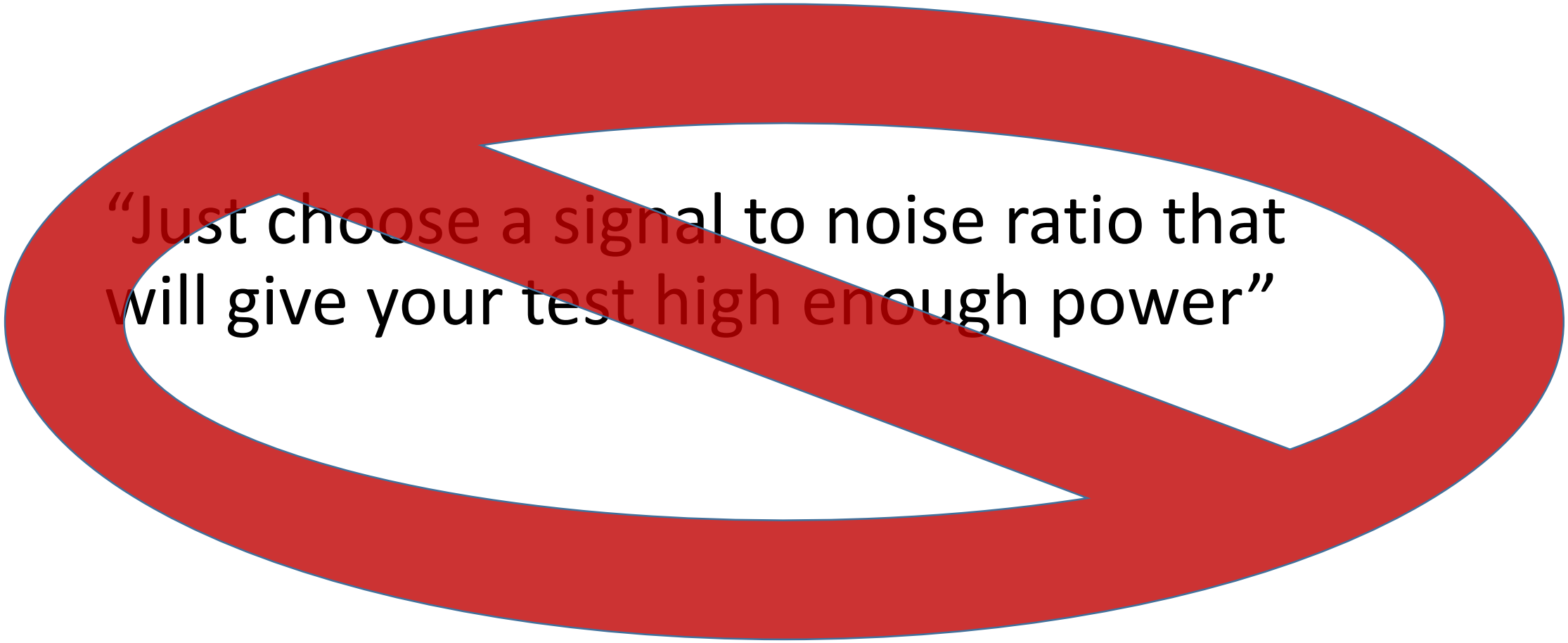
Aaron Ramert, STAT COE
aaron.ramert.ctr@afit.edu
www.AFIT.edu/STAT
937-255-3636 x 4573

DISTRIBUTION STATEMENT A. Approved for public release; distribution is unlimited. 88ABW-2019-2043

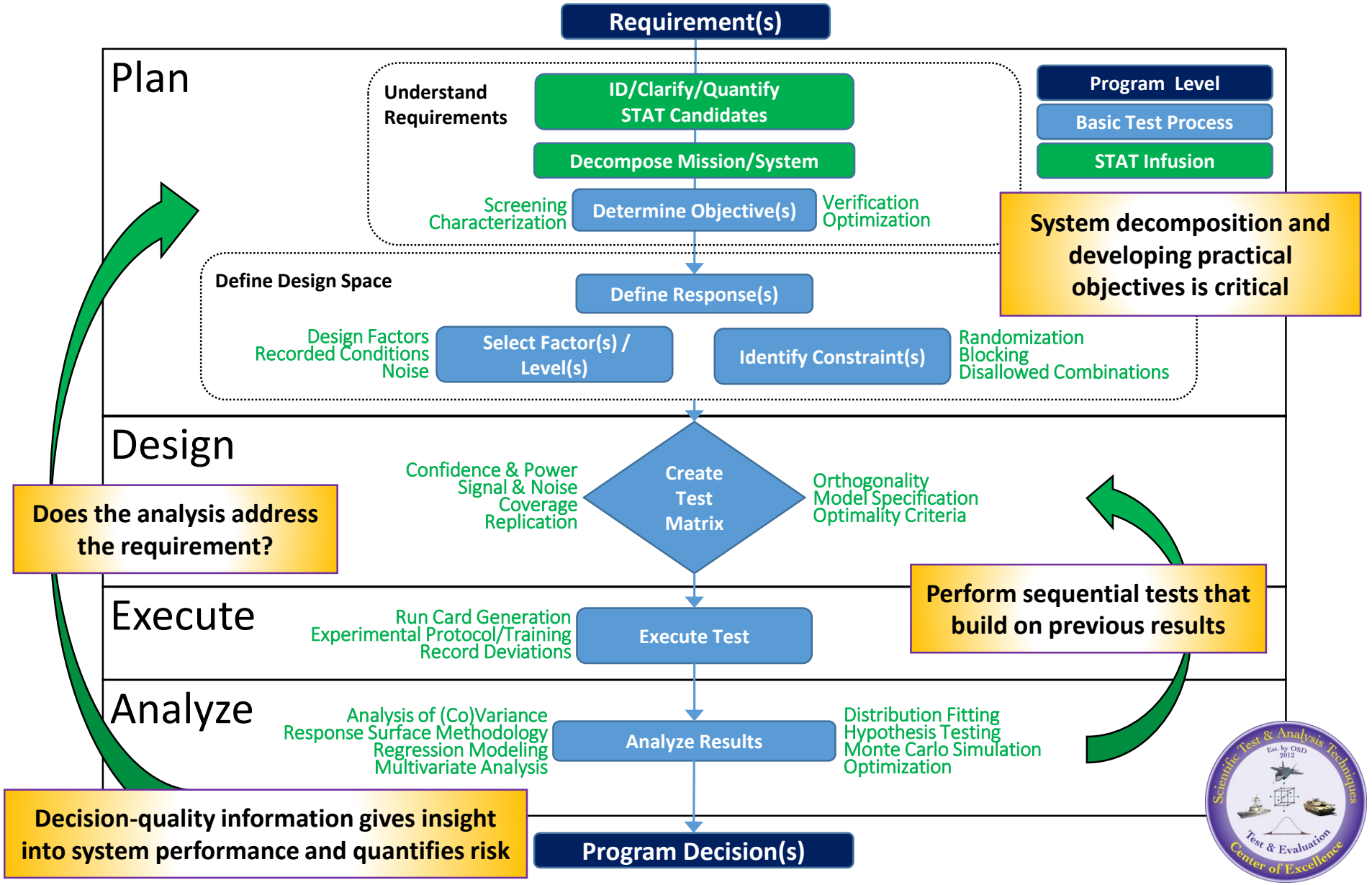
Agenda

- STAT Process
- Design of Experiments
- 5 Important Design Metrics used on DOE
- Signal Explained
- Noise Explained
- Improve the SNR
- Questions

Motivation for this brief



STAT in Test & Evaluation



Design of Experiments

- "A test or series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes that may be observed in the output response." (Montgomery 2009) Assign Causation
- To conserve resources, the difference to detect in the response should be set to determine the minimum practical change in system performance. – make δ as small as it has to be, but no smaller
- "The objective [of DOE] in many cases may be to develop a robust process, that is, a process affected minimally by external sources of variability." (Montgomery 2009) – Reduce the noise (σ)

Signal and Noise in the Test Process

INPUTS (Factors)

Attack Mode

Launch Angle off Nose

Ground Clutter

Delivery Altitude

Ground Range

Airspeed

Target Type

Uncontrollable factors

PROCESS:

Guided Bomb

OUTPUTS (Responses)

Range to Detect

Miss Distance

Impact Angle Error

Impact Velocity Error

δ

Noise

σ

Five Important Metrics used in DOE

Component	Effect of Increase	Effect of Decrease
Number of runs (n)	More runs is better! When n increases the power of the test increases.	With fewer runs (and no other changes) the power of the test will decrease.
Difference-to-Detect (δ)	When δ increases (and all else equal) the power increases	When δ decreases (and all else equal) the power decreases
System Noise (σ)	If the system has a high level of noise, it is more difficult to determine which portion of change in the response is due to the factor change and which portion is from noise.	A lower level of noise in a system makes it easier to determine the change in response due to a change in factor levels. This results in higher power and could lead to a smaller test.
Confidence ($1-\alpha$)	In cases where changing factor levels does not influence the response (H_0 is true) there is a higher probability we correctly conclude there is no influence.	In cases where changing factor levels does not influence the response (H_0 is true) there is a higher probability that we incorrectly conclude that it does have influence.
Power ($1-\beta$)	In cases where changing factor levels does have influence on the response (H_0 is false) there is a higher probability that we correctly conclude there is influence.	In cases where changing factor levels does have influence on the response (H_0 is false) there is a higher probability we incorrectly conclude there is no influence.

How do we **Determine** an Appropriate δ?

1. Understand the requirement. This may require critical thinking about what is and is not in the description.
2. Determine a practical change in system performance and relate it to a specific magnitude of change in response.
   1. A practical change is meaningful in a mission context and should elicit a decision from program leadership.
   2. If δ is too big we risk not detecting a practical change in performance
   3. If δ is too small we risk using excessive resources to detect a change in performance that we won't take action upon
3. How do we measure the response and how precise must the measurements be?

How do we **Calculate** an Accurate σ?

1. The best source of system noise is the Root Mean Square Error (RMSE) from a prior test.
2. You can use the RMSE from the test of a similar system. (conversion factor between systems?)
3. Calculate the sample standard deviation from observational data of the system.
4. In the absence of empirical data use subject matter expert (SME) opinion and the empirical rule; 95% will fall within two standard deviations of the mean of a normal distribution.

Why talk about δ and σ in terms of a ratio?

- Why talk about δ and σ in terms of a ratio?
  - It is a way to quantify the precision of a test
  - It is a way to compare the precision of different tests
- A signal to noise ratio of 2 means that we are trying to detect a factor level change that will move the response by at least two standard deviations.

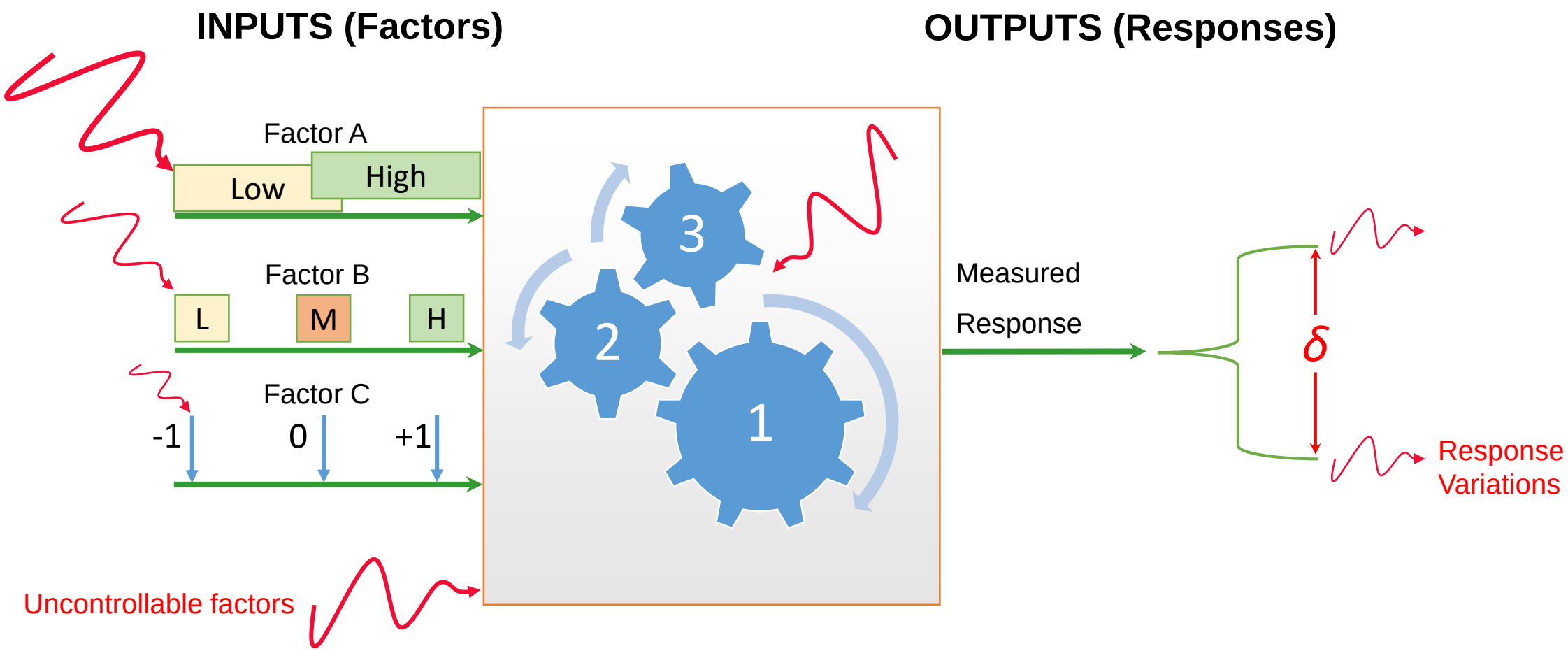
How do we make SNR better

- Can we adjust the difference-to-detect?
  - The proper δ should have been set to support the test objectives during planning.
  - If the test team changes δ they should be able to explain why the new value is acceptable.
    - New information or insight into the test
    - A change in the objective
    - New assumptions
- Changing δ should be one of the last metrics to tweak in test design and must be the result of a deliberate process

How do we make SNR better

- It may be possible and practical to reduce the variation in the system
- Input
  - Make factor levels as precise as possible
  - Have well defined procedures to reduce imprecision
- System
  - In DoD testing this is often a “black box”, we can not change anything
  - Reduce influences as much as possible
- Response
  - Reduce measurement error

Signal and Noise in the Test Process



Conceptual ANOVA

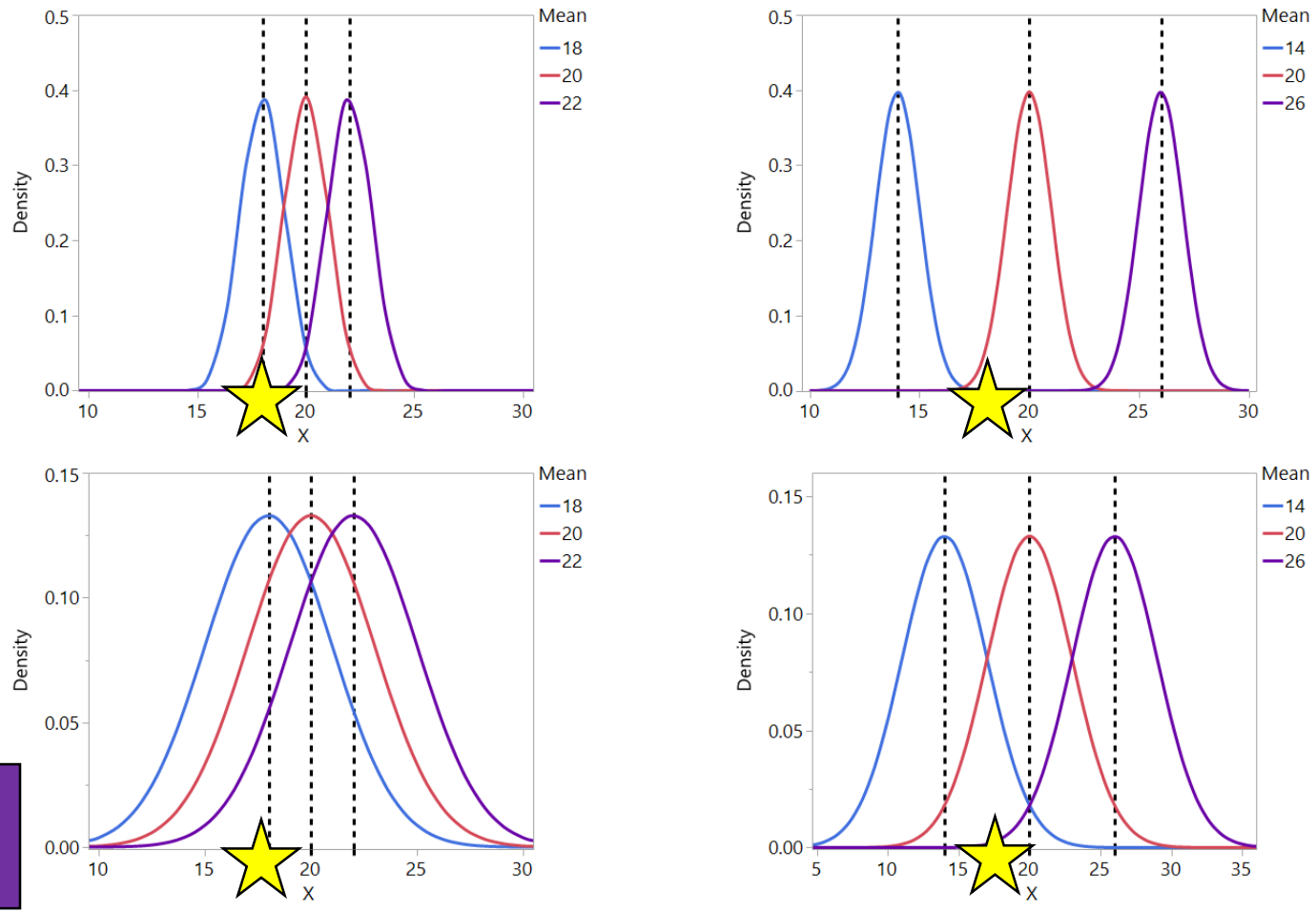
(One Factor/Three Levels)

Small — Difference in means — Large

σ small

Difference in noise

σ large



Easier to determine if data point is influenced by factor level

Difficult to determine if data point is influenced by factor level

Conclusion

- Any good test begins with a thorough understanding of the requirement(s) and deliberate planning
- Determine δ, make it as small as necessary, but no smaller
- Calculate σ by the best means available
- Talk SNR only after independently finding δ and σ
- It is okay to reduce σ, change δ only with a deliberate process and only when necessary

Full text is located at https://www.afit.edu/STAT/index.cfm under the tab "Best Practices and Test Planning Guides"

Questions?

STAT
RETURN OF THE SCIENTIFIC METHOD
COE